Excess noise for coherent radiation propagating through amplifying random media

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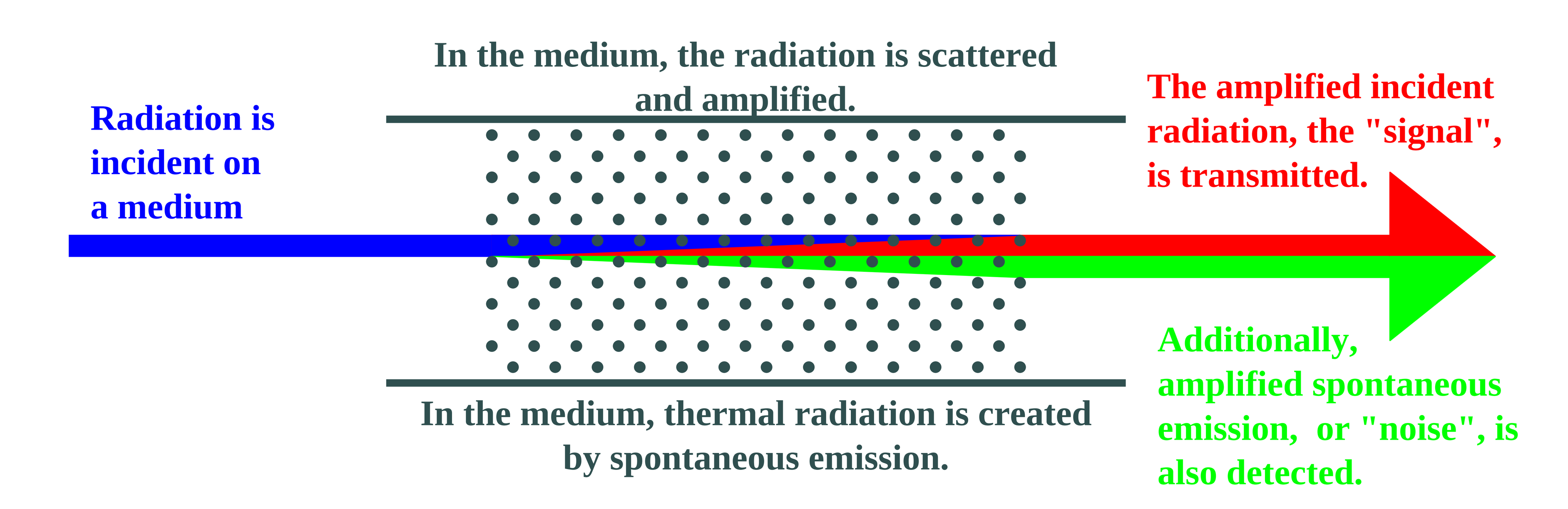

AIM: Theory of **photostatistics** for coherent radiation that has been amplified by a **random medium**

- expressed in terms of **scattering matrix**
- showing **beating** of coherent radiation with spontaneous emission
- increased **noise figure** due to inter-mode scattering
- contrast between amplifying and absorbing media

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Signal is characterised by: $\bar{I}$ Mean intensity
P Noise power

$\Rightarrow$ (Squared) signal-to-noise ratio $\frac{\bar{I}^2}{P}$

An amplifier can be described by its **noise figure**

$$F = \frac{\text{signal-to-noise ratio at the input}}{\text{signal-to-noise ratio at the output}}$$

The noise figure becomes characteristic for the amplifier if
- we use ideal detectors,
- an amplifying medium with complete population inversion,
- and consider strong amplification

Coherent radiation at the input:

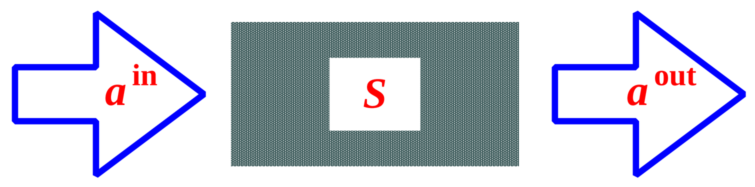
$P = \bar{I} \equiv I_0$ (only shot-noise)

At the output shot-noise plus **excess noise**:

$P = \bar{I} + P_{\rm exc}$

The excess noise is the beating of the incident radiation with the spontaneous emission. The pure amplified spontaneous emission can for a monochromatic input be filtered out.

We describe the system by its **scattering matrix** S



One can use the **input-output relation** $a^{\rm out} = S\, a^{\rm in}$ if the medium is neither amplifying nor absorbing (i.e. $S\,S^{+} = 1$). For an amplifying medium:

$$a^{\rm out} = S\, a^{\rm in} + V\, c^{+}$$

Bosonic commutation relations give the condition

$$S\,S^{+} + V\,V^{+} = 1$$

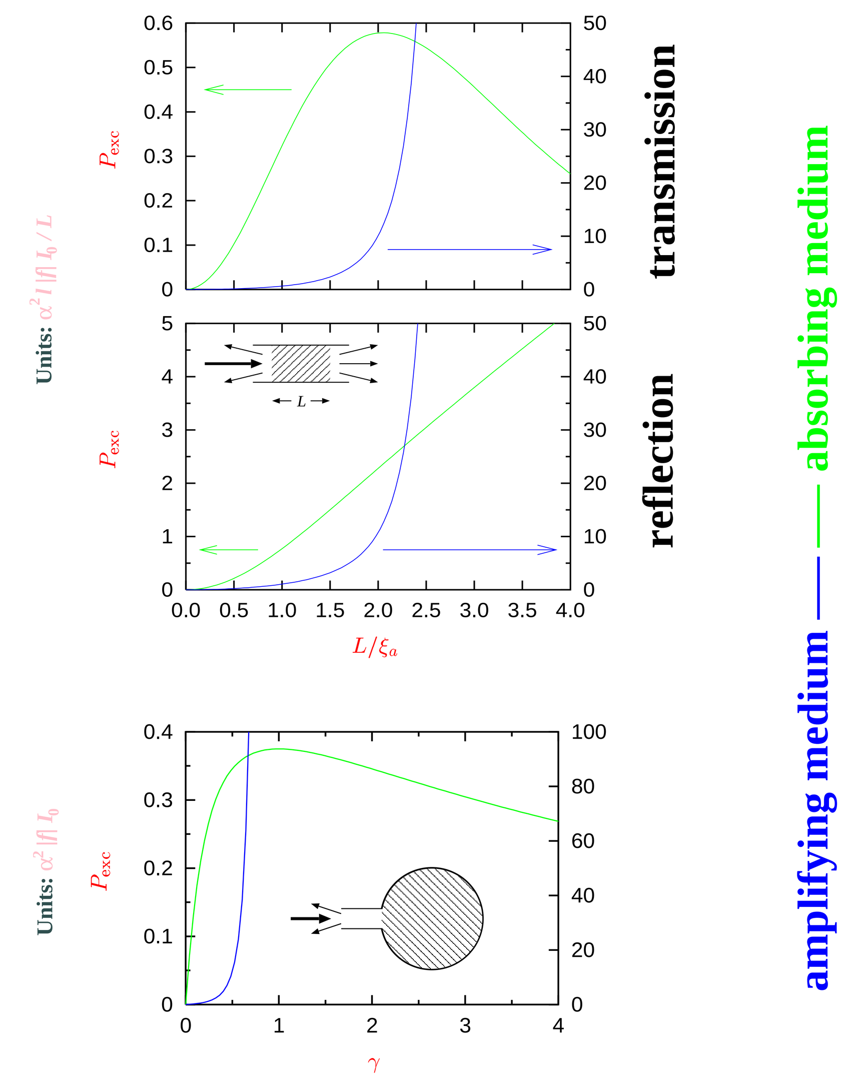
The operators c have the average

$$\langle c_n(\omega), c^{+}_m(\omega') \rangle = -\,\delta_{nm}\,\delta(\omega - \omega')\,f(\omega, T)$$

(f is the **Bose-Einstein function**, evaluated at negative temperature, $T<0$)

[1] J. R. Jeffers, N. Imoto, and R. Loudon, Phys. Rev. A 47, 3346 (1993)
[2] R. Matloob, R. Loudon, S. M. Barnett, and J. Jeffers, Phys. Rev. A 52, 4823 (1995)
[3] T. Gruner and D.-G. Welsch, Phys. Rev. A 54, 1661 (1996)

Results: Excess noise



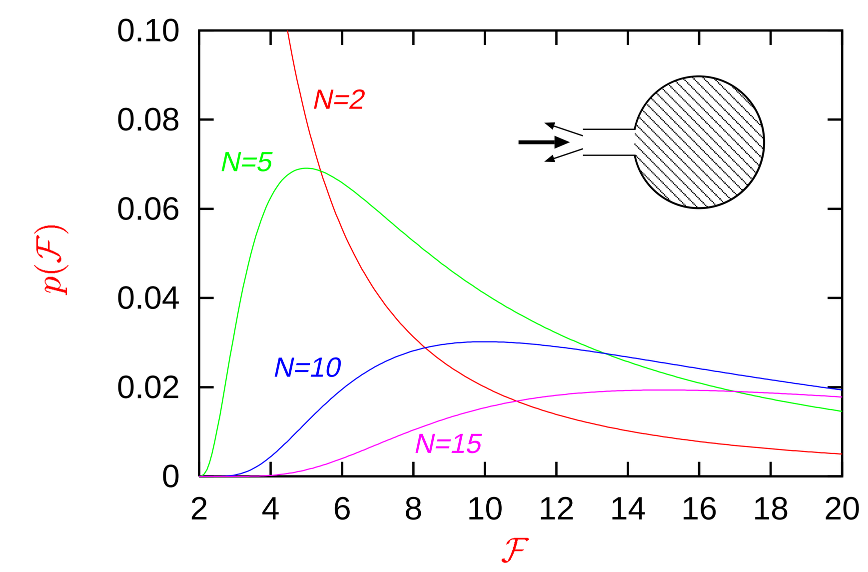
For certain geometries, the curves have a **maximum** because:
- small absorption: no beating
- large absorption: no signal

Results: Distribution of F

Random-matrix theory allows to compute distribution of the noise-figure of a chaotic optical cavity for **arbitrary** number of modes N in the waveguide at **laser threshold**.

$$P(F) = -2f(N-1)\left(1 + \frac{2f}{F}\right)^{N-2} F^{-2}$$

$\Rightarrow$ **Most likely value of F is N, moments diverge.**



General theorem by Caves [1]:
For any linear amplifier, $F \geq 2$.

Earlier theories on amplification of radiation consider only **1 D-amplifiers**
$\Rightarrow$ (e.g.) the group of Loudon [2], [3]
These theories confirm the lower bound $F = 2$

Problem: Random media are intrinsically **3 D**
- more than one propagating mode in the amplifier
- existance of inter-mode scattering

Three dimensionality also important for describing a **random laser** [4].

[1] C. M. Caves, Phys. Rev. D 26, 1817 (1982)
[2] J. R. Jeffers, N. Imoto, and R. Loudon, Phys. Rev. A 47, 3346 (1993)
[3] R. Matloob, R. Loudon, M. Artoni, S. Barnett, and J. Jeffers, Phys. Rev. A 55, 1623 (1997)
[4] D. Wiersma and A. Lagendijk, Physics World, Jan. 1997, p. 33

Results for a measurement in **transmission**, over a time τ, with **detector efficiency** α, radiation incident in mode m_0:

$$\bar{I} = \alpha I_0 \left(t^\dagger t\right)_{m_0 m_0}$$

$$P_{\rm exc} = 2\alpha^2 f I_0 [t^\dagger \left(1 - rr^\dagger - tt^\dagger\right) t]_{m_0 m_0}$$

Arbitrary **cumulants** of the photocount:

$$\kappa_k = k!\alpha^k \tau f^{k-1} I_0 [t^\dagger \left(1 - rr^\dagger - tt^\dagger\right)^{k-1} t]_{m_0 m_0}$$

Noise figure for **strong** amplification:

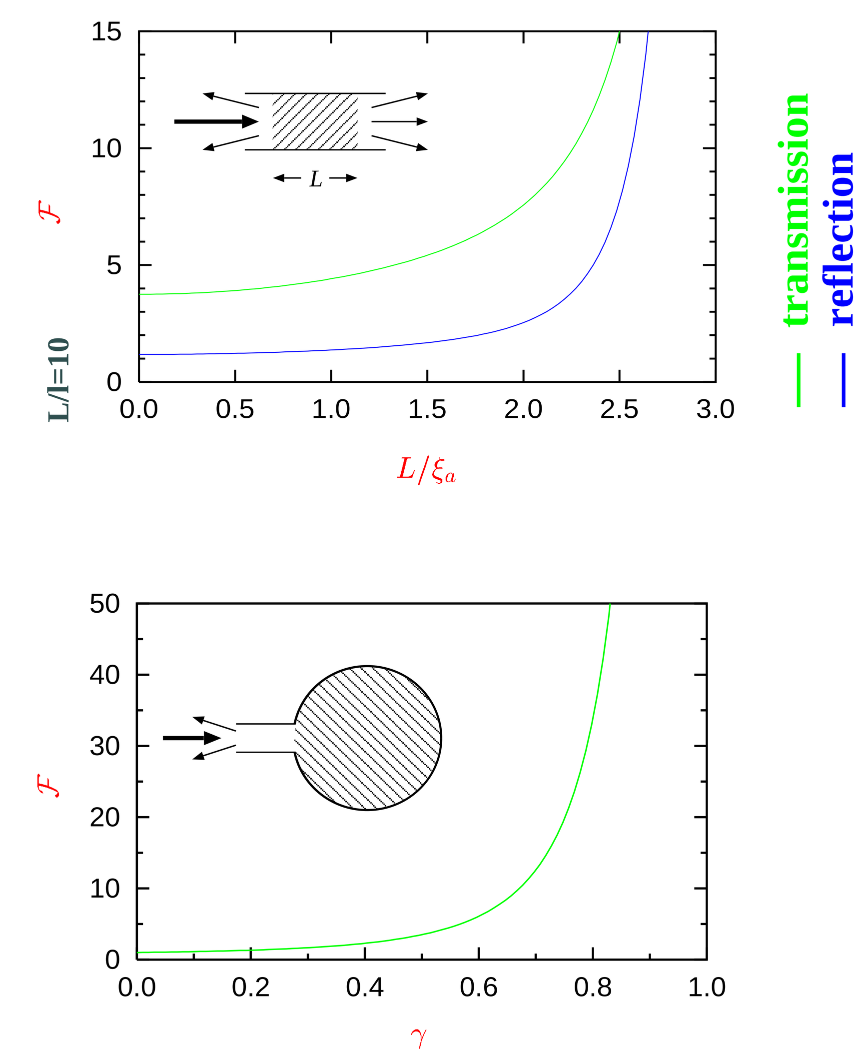
$$F = -2f \frac{\left(t^\dagger r r^\dagger t + t^\dagger t t^\dagger t\right)_{m_0 m_0}}{(t^\dagger t)^2_{m_0 m_0}}$$

Noise figure is **minimal** ($F=2$ for $f=-1$) only if
- no reflection: $rr^\dagger = 0$
- no inter-mode scattering: $t^\dagger t$ diagonal

$\Rightarrow$ **Noise figure of a random medium usually much larger than minimal value!**

Similar for a measurement in **reflection**.

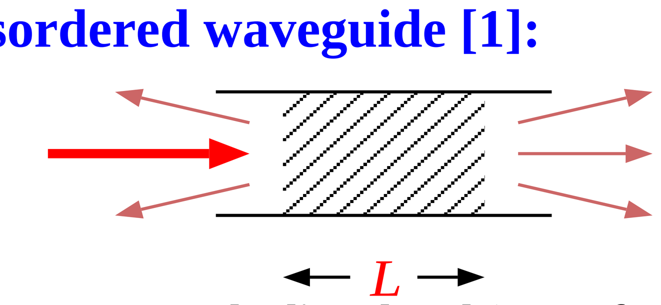
Results: Mean noise figure



$\Rightarrow$ **Mean noise figure diverges at laser threshold.**

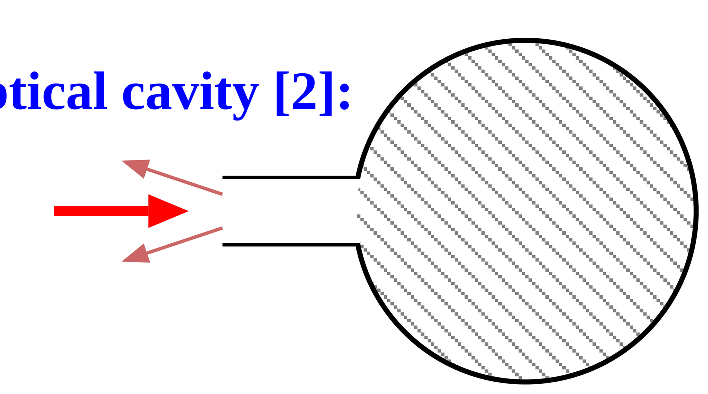
The studied models

Disordered waveguide [1]:



- strongly disordered (mean free path l)
- weakly amplifying / absorbing
- many propagating modes
- parameter: L / ξ_a (ξ_a is the absorption length)

Optical cavity [2]:



- many modes in adjacent waveguide
- parameter: γ (ratio absorption time over amplification time)

[1] P. W. Brouwer, Phys. Rev. B 57, 10526 (1998)
[2] C. W. J. Beenakker, Phys. Rev. Lett. 81, 1829 (1998)