Lasing Treshold of Random Lasers

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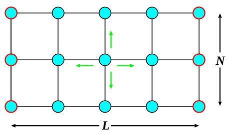
We compute the distribution of the decay rates of the eigenstates in a random laser from a numerical model. From the results of the numerical simulations, we are able to find simple analytical formulae that describe those results well. We use the decay rates to calculate the lasing threshold of random lasers. In addition, we analyse the influence of spatial correlations.

We compute the lasing threshold via the eigenmodes and eigenvalues of the **Hamiltonian**. The real part of the **eigenvalue** gives the eigenfrequency, the imaginary part gives the **decay rate**.

By generating thousands of samples with different realisation of the disorder, a histogram of the decay rates is computed. The **histograms** for different system parameters are **fitted** to functional forms.

We analysed the fitting parameters as a function of the system parameters and were able to find simple relations. In this way, we arrive at **explicit expressions** for the decay rate distribution for arbitrary system parameters.

The **Anderson Hamiltonian** models transport by nearest-neighbour hopping with rate 1. On each lattice site, there is a spatially varying potential. The outcoupling at both ends of the sample is described by an imaginary term. The sample length is L, the sample width is N.



$$\mathcal{H}_{(x,y),(x',y')} = \delta_{xx'}\delta_{yy'} \times \left[P(x,y) - i(\delta_{1y} + \delta_{Ly})\right] + \delta_{yy'}\left(\delta_{x+1,x'} + \delta_{x-1,x'}\right) + \delta_{x,x'}\left(\delta_{y+1,y'} + \delta_{y-1,y'}\right)$$

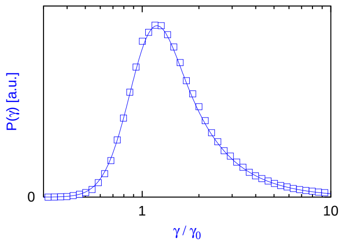
A disordered medium is modelled by assigning random values to $P(x,y)$.

In the theory of disordered media, two important **regimes** are distinguished.

- In the diffusive regime, eigenstates are extended and efficient transport is possible. A sample is diffusive for weak disorder and/or short sample length.
- In the localised regime, eigenstates falls off exponentially in space and transport is strongly inhibited. Samples are localised if the disorder is strong.

In the following, we will discuss these two regimes separately as they differ in many aspects.

Having computed the numerical histograms of the decay rate distributions for the individual modes, we **fit** them to some educated guesses for the shape of the distribution functions. We need to distinguish between the diffusive and the localised regimes.



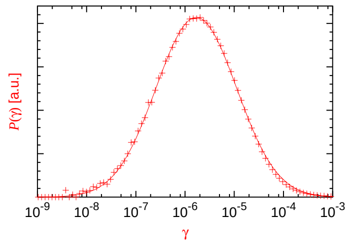
Eigenstates are extended also in chaotic cavities. For the diffusive regime we thus start from the known decay rate distribution for chaotic cavities and rescale it:

$$P(\gamma) \propto \frac{1}{\gamma^2}\left[1 - \frac{\Gamma(M+1, M\gamma/\gamma_0)}{\Gamma(M+1)}\right]$$

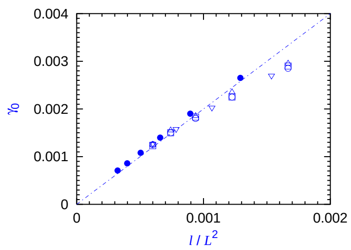
(For a chaotic cavity, M gives the size of the opening of the cavity.)

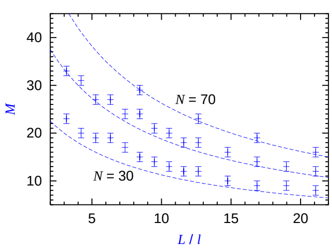
A large number of random contributions are superimposed in a disordered medium, usually resulting in normal (=Gaussian) distributions. Since in the localised regime all eigenstates fall off exponentially, this rather yields a log-normal distribution:

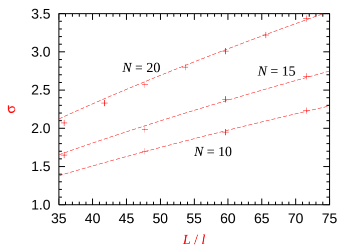
$$P(\gamma) \propto \exp\left[-\frac{(\log\gamma - \log\gamma_0)^2}{\sigma^2}\right]$$

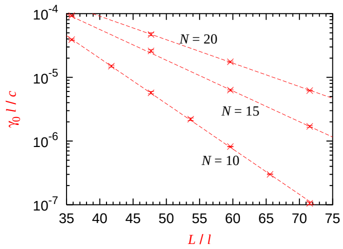


As the two figures above show, the numerically computed histograms follow the analytic forms well. The parameter γ_0 gives the typical decay rate, for both the diffusive regime and the localised regime. The width (and shape) is determined by M and σ, respectively. The dependence of γ_0, M and σ on the system parameters are:









The numerically computed values for γ_0, M and σ are well described by the following analytic expressions (shown as dashed lines above), depending only on the sample length L, the sample width N and the mean free path l.

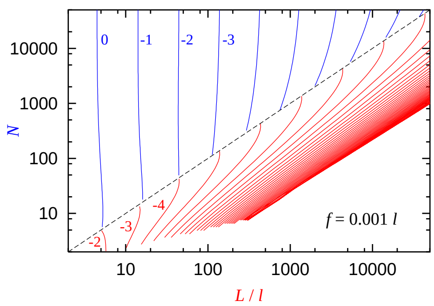
$$\gamma_0 = \frac{2cl}{L^2} \qquad M = \frac{N}{1 + L/(6l)} \qquad \sigma = \frac{2}{3}\left(\frac{2L}{[N+1]al}\right)^{2/3} \qquad \gamma_0 = \frac{a}{N^2}\exp\left(-\frac{2L}{[N+1]al}\right) \quad \text{with} \quad a = 1.12\,.$$

An eigenmode starts to lase if photons in it escape (=decay) more slowly than they are replenished by pumping. The **lasing threshold** is thus given by the **smallest decay rate** of all eigenmodes.

The distribution of the smallest element γ_L out of a set of K random values can be computed from the distribution $P(\gamma)$ determined above. The most likely lasing threshold γ_L follows from

$$0 = \frac{dP(\gamma_L)}{d\gamma_L}\left[1 - \int_0^{\gamma_L} P(\gamma')d\gamma'\right] - (K-1)[P(\gamma_L)]^2\,.$$

The solutions of this equation are depicted on the right, for $P(\gamma)$ both from the diffusive and localised regime. (The properties of the laser dye enter via the quantity f, $K = fNL/\Delta$. The numbers x on the lines mean 10^x.)



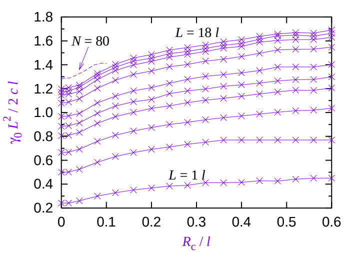
The lasing threshold γ_L is dominated by the typical decay rate γ_0. The small-γ tail of $P(\gamma)$ does not contain enough weight to allow a lasing threshold that is significantly smaller than γ_0 (unless K becomes exponentially large).

This means that for the diffusive regime, the lasing threshold is basically determined by the length of the sample alone whereas in the localised regime both the length and the width of the sample are important.

Localised samples offer the advantage of a smaller decay rate at given length but must be limited in width, making an experimental realisation more difficult. Diffusive samples, on the other hand, can be made very long since their width does not need to be restricted.

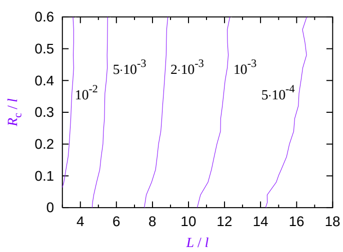
Many experimental results suggest that the lasing mode is localised while a direct experimental analysis of the sample shows that it is in the diffusive regime. It seems that there can be a few localised modes (which become the lasing modes) in an otherwise diffusive sample.

Only one explanation has been offered so far. It was suggested that localised modes could be created and the decay rates be decreased if the potential $P(x,y)$ becomes spatially correlated within some correlation radius R_c.



As the figure on the left shows, the typical decay rate increases as the correlation radius R_c is increased. The behaviour of the **average** mode thus is opposite to the prediction.

The figure on the right shows the computed lasing threshold. In contrast to the figure on the left, these data describe a **special** mode with lower-than-average decay rate, namely the lasing mode. Also here, an increase of the rate is seen when spatial correlations are introduced. No indication for the formation of localised modes is found.



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